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SIMULTANEOUS TESTS FOR THE EQUALITY OF COVARIANCE MATRICES AGAINST CERTAIN ALTERNATIVES

P. R. KRISHNAIAH
APPLIED MATHEMATICS RESEARCH LABORATORY

Project No. 7071

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Page 6, line 2: $|w - v|$ should read as $|w - v| > 1$

Page 8: Add "for $j = 1, 2, \ldots, p$" above last line

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FOREWORD

This report was prepared for the Applied Mathematics Research Laboratory, Aerospace Research Laboratories by Dr. P. R. Krishnaiah under Project 7071, "Research in Applied Mathematics". It contains some procedures for testing the hypothesis of equality of covariance matrices against different alternatives when the underlying populations are multivariate normal.

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ABSTRACT

In this paper, we consider the problems of testing for the equality of covariance matrices against certain alternatives when the underlying populations are multivariate normal. The alternative hypotheses considered are (i) at least one covariance matrix is not equal to the covariance matrix of the next population, (ii) at least one covariance matrix is not equal to the covariance matrix of the standard population and (iii) at least one covariance matrix is not equal to the covariance matrix of another population.

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1. Introduction and Summary

In many situations, it is of interest to test for the equality of variances or covariance matrices against certain alternatives. Hartley [6] considered the problem of testing for the equality of variances against the alternative that at least one variance is different from the other. Gnanadesikan [3] considered the problem of testing for the equality of variances against the alternative that at least one variance is not equal to the standard. Recently, Krishnaiah [11] considered testing for the equality of variances against the alternative that at least one variance is not equal to the next. In the above procedures, it was assumed that the underlying populations are univariate normal. In this paper, we consider multivariate generalizations of the above test procedures. The test procedures proposed in this paper are based upon expressing the total hypothesis as the intersection of some elementary hypotheses and testing these elementary hypotheses by using conditional distributions. In the two sample case, our procedures are similar (but not equivalent) to the procedure proposed by Roy [14]; the test statistics used by him in testing some of the elementary hypotheses are different from those used in this paper.

2. Preliminaries and Statement of Problems

Let $S_i = (s_{iqr})$ denote ith sample sums of squares and cross products (SP) matrix and let $n_i + 1$ denote ith sample size. Let Σ_{ij} denote top jxj left hand corner of $\Sigma_i = (\sigma_{iqr})$ and let S_{ij} denote the top jxj left hand corner of $S_i = (s_{iqr})$. Also, let $S_{ij}^{-1} = (s^*_{ijtu})$ and let σ^2_{0j} denote the common value of σ^2_{ij} (defined below) when

$\sigma^2_{1j} = \ldots = \sigma^2_{kj}$.

In addition,

$$\underset{\sim}{\beta}_{ij} = \begin{pmatrix} \beta_{ij1} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{ijj} \end{pmatrix} = \Sigma_{ij}^{-1} \begin{pmatrix} \sigma_{i1,\, j+1} \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{ij,\, j+1} \end{pmatrix}$$

$$\underset{\sim}{b}_{ij} = \begin{pmatrix} b_{ij1} \\ \cdot \\ \cdot \\ \cdot \\ b_{ijj} \end{pmatrix} = S_{ij}^{-1} \begin{pmatrix} s_{i1,\, j+1} \\ \cdot \\ \cdot \\ \cdot \\ s_{ij,\, j+1} \end{pmatrix}$$

$$s^2_{i\,,\,j+1} = \frac{|S_{i,\, j+1}|}{|S_{ij}|}, \quad \sigma^2_{i,\, j+1} = \frac{|\Sigma_{i,\, j+1}|}{|\Sigma_{ij}|} \text{ for } j=1, 2, \ldots, (p-1)$$

$$s^2_{i1} = s_{i11}, \qquad \sigma^2_{i1} = \sigma_{i11} \quad , \quad s^2_{\cdot\,,j+1} = \sum_i s^2_{i,\, j+1} \quad .$$

$$D_{imtu} = (b_{itu} - b_{mtu})^2 \Big/ (s^*_{ituu} + s^*_{mtuu}), \qquad N = \sum_{i=1}^{k} n_i$$

$$F_{imt} = \frac{s^2_{it}\,(n_m - t+1)}{s^2_{mt}\,(n_i - t+1)} \quad , \quad F_{imtu} = \frac{(N-kt)\, D_{imtu}}{s^2_{\cdot\,,\,t+1}} \quad .$$

Also, let

$$H_{j1}: \sigma^2_{1j} = \ldots = \sigma^2_{kj} \qquad H_{j2}: \beta_{1j} = \ldots = \beta_{kj}$$

$$A_{1j1} = \bigcup_{i=1}^{k-1} [\, \sigma^2_{ij} \neq \sigma^2_{i+1,j} \,] \qquad A_{1j2} = \bigcup_{i=1}^{k-1} [\, \beta_{ij} \neq \beta_{i+1,j} \,]$$

$$A_{2j1} = \bigcup_{i=1}^{k-1} [\, \sigma^2_{ij} \neq \sigma^2_{kj} \,] \qquad A_{2j2} = \bigcup_{i=1}^{k-1} [\, \beta_{ij} \neq \beta_{kj} \,]$$

$$A_{3j1} = \bigcup_{i \neq i'=1}^{k} [\, \sigma^2_{ij} \neq \sigma^2_{i'j} \,] \qquad A_{3j2} = \bigcup_{i \neq i'=1}^{k} [\, \beta_{ij} \neq \beta_{i'j} \,]$$

In this paper, we consider the problem of testing H against A_1, A_2 and A_3 where $H: \Sigma_1 = \ldots = \Sigma_k$, $A_1 = \bigcup_{j=1}^{p} A_{1j1} \bigcup_{j=1}^{p-1} A_{1j2}$, $A_2 = \bigcup_{j=1}^{p} A_{2j1} \bigcup_{j=1}^{p-1} A_{2j2}$ and $A_3 = \bigcup_{j=1}^{p} A_{3j1} \bigcup_{j=1}^{p-1} A_{3j2}$

The test procedures considered in this paper are based on the following method. We first test H_{11} against the alternative of interest. If H_{11} is rejected, we declare that H is rejected. If H_{11} is accepted, we proceed further and test H_{21} and H_{12} holding the first variate fixed. If $H_{12} \cap H_{21}$ is accepted, we proceed further and test H_{31} and H_{32} holding the second variate fixed. We continue this proceudre until H is accepted or rejected. Here we note that $\bigcap_{j=1}^{r} H_{j1} \bigcap_{j=1}^{r-1} H_{j2}$ is equivalent to the hypothesis that

$$\Sigma_{1r} = \ldots = \Sigma_{kr}.$$

We need the following known results (see [14]) in the sequel:

When S_{ij} is fixed, the distribution of $\underset{\sim}{b}_{ij}$ is independent of the distribution of $s^2_{i,j+1}$; the distribution of $\underset{\sim}{b}_{ij}$ is j-variate normal with mean vector β_{ij} and covariance matrix

$\sigma^2_{i,\;+1} S^{-1}_{ij}$, and $s^2_{i,j+1}/\sigma^2_{i,j+1}$ is distributed as χ^2 with (n_i-j) degrees of freedom.

3. Test for H Against A_1

The following lemma is needed in the sequel.

Lemma 4.1

If $x_1, x_2, \ldots, x_k$ are distributed independently as central chi-square variates with $m_1, m_2, \ldots, m_k$ degrees of freedom, then

$$f(F_{12}, F_{23}, \ldots, F_{k-1,k}) = \frac{(m_1/m_k)\left[\prod_{j=1}^{k-1} m_j \prod_{i=j}^{k-1} F_{i,i+1}/m_k\right]^{(m_k-2)/2}}{\prod_{j=1}^{k} \Gamma(m_j/2)\left[1+\frac{1}{m_k}\sum_{j=1}^{k-1} m_j \prod_{i=j}^{k-1} F_{i,i+1}\right]^{\Sigma m_j/2}}$$

where $F_{ij} = \dfrac{x_i \; m_i}{x_j \; m_i}$

The proof of the above lemma is given in [11].

We will first consider the problem of testing H_{j1} against the alternative A_{1j1} when the first (j-1) variates are held fixed (with the understanding that no variate is held fixed when H_{11} is tested). In this case, we accept H_{j1} if and only if

$$\lambda_{ij} \le F_{i,i+1,j} \le \mu_{ij} \tag{4.1}$$

where λ_{ij} and μ_{ij} are chosen such that

$$P[\lambda_{ij} \le F_{i,i+1,j} \le \mu_{ij};\ i = 1, 2, \ldots, k-1 \mid H_{j1}] = P_j \tag{4.2}$$

When H_{j1} is true, $s^2_{1j}/\sigma^2_{0j}, \ldots, s^2_{kj}/\sigma^2_{0j}$ are independently distributed as chi-square

variates with $(n_1-j+1),\ldots,(n_k-j+1)$ degrees of freedom. So, using Lemma 4.1, we can write down the joint distribution of $F_{12}, F_{23},\ldots,F_{k-1,k}$ when H_{j1} is true. We will now discuss about a procedure for testing H_{j2} against A_{1j2} when H_{j1} is true and when the first j variates are held fixed.

When H_{j1} is true and the first j variates are held fixed, we accept H_{j2} if and only if

$$(4.3) \qquad F_{i,i+1,ju} \le c_{ij}, \qquad u = 1,2,\ldots,j \quad i = 1,2,\ldots,k-1$$

where c_{ij}'s are chosen such that

$$(4.4) \qquad P[F_{i,i+1,ju} \le c_{ij};\ u = 1,2,\ldots,j, i = 1,2,\ldots,k-1 \mid H_{j1} \cap H_{j2}] = P_j'.$$

When H_{j1} is true, $s^2_{..j+1}/\sigma^2_{0,j+1}$ is distributed as a chi-square variate with $(N-kj)$ degrees of freedom and it is distributed independently of $D_{i,i+1,ju}$ for $i = 1,2,\ldots,k-1$ and $u = 1,2,\ldots,j$. Also, when $H_{j1} \cap H_{j2}$ is true, the joint distribution of

$$(D_{12j1},\ldots,D_{k-1,kj1}, D_{12j2},\ldots, D_{k-1,kj2},\ldots, D_{12jj},\ldots, D_{k-1,kjj})$$

is a central multivariate chi-square distribution with 1 degree of freedom and with Ω^j as the covariance matrix of the "accompanying" multivariate normal where

$$\Omega^j = \begin{bmatrix} \Omega^j_{11} & \Omega^j_{12} & \cdots & \Omega^j_{1j} \\ \Omega^j_{21} & \Omega^j_{22} & \cdots & \Omega^j_{2j} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \Omega^j_{j1} & \Omega^j_{j2} & \cdots & \Omega^j_{jj} \end{bmatrix}$$

$$\Omega^j_{ii'} = (\omega^j_{ii'vw})\,\sigma^2_{0,j+1},$$

$$\omega^j_{ii'vw} = \begin{cases} 0 & |w-v| > \\[2ex] \dfrac{(s^*_{vjii'} + s^*_{v+1,jii'})}{\sqrt{(s^*_{vjii} + s^*_{v+1,jii})(s^*_{vji'i'} + s^*_{v+1,ji'i'})}} & w = v \\[3ex] \dfrac{-s^*_{tjii'}}{\sqrt{(s^*_{vjii} + s^*_{v+1,ii})(s^*_{wji'i'} + s^*_{w+1,ji'i'})}} & |w-v| = 1 \end{cases}$$

and $t = \max.(w, v)$; (for the definition of the "accompanying" multivariate normal, see [7]). So, the joint distribution of

$$(F_{12j1}, \ldots, F_{k-1,kj1}, \ldots, F_{12jj}, \ldots, F_{k-1,kjj})$$

is a multivariate F distribution with $(1, N-kj)$ degrees of freedom and with Ω^j as the covariance matrix of the "accompanying" multivariate normal. For various details on the multivariate F distribution, the reader is referred to [9, 10].

Now, combining (4.1), (4.2), (4.3) and (4.4) we use the following procedure for testing H against A_1.

Accept H against A_1 if and only if

$$(4.5)\qquad \begin{cases} \lambda^*_{ij} \le F_{i,i+1,j} \le \mu^*_{ij} & i = 1, 2, \ldots, k-1 \quad j = 1, 2, \ldots, p \\[3ex] F_{i,i+1,ju} \le c^*_{ij} & u = 1, 2, \ldots, j \quad j = 1, 2, \ldots, (p-1) \\ & i = 1, 2, \ldots, k-1 \end{cases}$$

where $\lambda^*_{ij}, \mu^*_{ij}$ and c^*_{ij} are chosen such that the probability of (4.5) holding good, when H is true, is $(1-\alpha)$. But this probability is equal to $\prod_{j=1}^{p} q_j \prod_{j=1}^{p-1} q'_j$ where

$$q_j = P[\lambda^*_{ij} \le F_{i,i+1,j} \le \mu^*_{ij};\ i=1,2,\ldots,k-1 \mid H_{1j}]$$

$$q'_j = P\left[F_{i,i+1,ju} \le c^*_{ij};\ u=1,2,\ldots,j \quad \begin{array}{l} j=1,2,\ldots,(p-1) \\ i=1,2,\ldots,(k-1) \end{array} \Big| H_{2j}\right].$$

The optimum choice of the critical values is not known. For practical purposes, we impose the following restrictions.

$$q_1 = \ldots = q_p = q'_1 = \ldots = q'_{p-1} = (1-\alpha)^{1/2p-1}$$

$$c^*_{ij} = c^*_j .$$

In addition, we impose the restriction that the test associated with testing H_{1j} is locally unbiased.

The $(1-\alpha)$ % simultaneous confidence intervals associated with the above test procedure are given by

$$\frac{\lambda^*_{ij}\, s^2_{i+1,j}(n_i-j+1)}{s^2_{ij}\,(n_{i+1}-j+1)} \le \frac{\sigma^2_{i+1,j}}{\sigma^2_{ij}} \le \mu^*_{ij} \frac{(n_i-j+1)}{(n_{i+1}-j+1)} \frac{s^2_{i+1,j}}{s^2_{ij}}$$

$$i=1,2,\ldots,k-1 \quad j=1,2,\ldots,p$$

$$\left|b_{iju} - b_{i+1,ju} - \beta_{iju} + \beta_{i+1,ju}\right| \le \frac{c^*_{ij}\, s^2_{\psi j+1}\,(s^*_{ijuu} + s^*_{i+1,juu})}{(N-kj)}$$

$$u=1,2,\ldots,j \quad j=1,\ldots,(p-1)$$

$$i=1,2,\ldots,k-1 .$$

4. Tests for H Against A_2 and A_3.

When H is tested against A_2, we accept H if and only if

$$a_{ij} \le F_{ikj} \le b_{ij} \qquad i = 1,2,\ldots,k-1 \qquad j = 1,2,\ldots, p$$

$$F_{ikju} \le c_{ij} \qquad u = 1,2,\ldots, j \qquad j = 1,2,\ldots, p$$
$$i = 1,2,\ldots, k-1$$

where a_{ij}, b_{ij}, c_{ij} are chosen such that

$$\prod_{j=1}^{p} Q_j \prod_{j=1}^{p-1} Q'_j = 1-\alpha ,$$

and

$$Q_j = P[a_{ij} \le F_{ikj} \le b_{ij} \; ; i = 1,2,\ldots, k-1 \;\; j = 1,2,\ldots, p \,|\, H_{1j}]$$

$$Q'_j = P[F_{ikju} \le c_{ij;} \;\; u = 1,2,\ldots, j \;\; i = 1,2,\ldots, k-1 \quad H_{2j} \;] .$$

We can evaluate $Q_1, \ldots, Q_p$ by using the methods (or their modifications) discussed in [1, 4, 5, 8, 12] whereas $Q'_1, \ldots, Q'_{p-1}$ can be evaluated by using the methods discussed in [9, 10]. The optimum choice (in terms of increasing power of the test) of the critical values is not known. But, for practical purposes, we can choose them by imposing restrictions similar to those in the previous section.

We will now propose a procedure to test H against A_3 when the sample sizes are equal to $(n+1)$. According to this procedure, we accept H if and only if

$$\begin{cases} 1/\lambda_j \le F_{ii'j} \le \lambda_j ; \; i \ne i' = 1,2,\ldots, k \\ \qquad F_{ii'ju} \le c_j ; \; i \ne i' = 1,2,\ldots, k \quad u = 1,2,\ldots, j \end{cases}$$

where

$$\prod_{j=1}^{p} R_j \prod_{j=1}^{p-1} R'_j = 1-\alpha$$

$$R_j = P[\,1/\lambda_j \le F_{ii'j} \le \lambda_j;\ i \ne i' = 1, 2, \ldots, k \,|\, H_{1j}]$$

$$R'_j = P[\,F_{ii'ju} \le c_j;\ i \ne i' = 1, 2, \ldots, k \quad u = 1, 2, \ldots, j \,|\, H_{2j}]\,.$$

Using the method discussed in [6], we can evaluate $R_1, \ldots, R_p$. In order to evaluate $R'_1, \ldots, R'_{p-1}$ we note that, when H_2 is true and j is fixed, the statistics $F_{ii'ju}$ are jointly distributed as a singular multivariate F distribution. So it is complicated to obtain exact values of R'_j. But, we can obtain approximate values of R'_j by using Bonferroni's inequalities [2; p. 100]. For practical purposes, we can choose the critical values such that

$$R_1 = \ldots = R_p = R'_1 = \ldots = R_{p-1} = (1-\alpha)^{1/2p-1}$$

The simultaneous confidence intervals associated with the above test procedures can be obtained easily.

5. General Remarks

Roy [14] proposed a procedure, based on conditional distributions, for testing the equality of two covariance matrices. But, the lengths of the confidence intervals associated with the procedures proposed in this paper are at least as short as the lengths of the corresponding confidence intervals associate with the procedure by Roy [14]. In the univariate case, the procedures proposed in this paper for testing H against A_1, A_2 and A_3 are respectively equivalent to the procedures considered by Krishnaiah [11], Gnanadesikan [3] and Hartley [6].

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13. ABSTRACT

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